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Semiclassical strings probing NS5 brane wrapped on S^5

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Abstract: We study little string theory on $R^1 \times S^5$, defined by a theory which lives on type IIA N NS5 branes wrapped on S^5, using its supergravity dual. In particular we study semiclassical rotating closed strings in this background. We also consider Penrose limit of this background that leads to a plane wave on which string theory is exactly solvable.

Keywords: Penrose limit and pp-wave background, AdS-CFT Correspondence.

Contents

1. Introduction

Little string theory is a nonlocal theory that has many features of string theory such as T-duality, though it is a non-gravitational theory [1 − 3]. It is the theory which lives on the worldvolume of type IIA NS5 brane in its decoupling limit. From holographic point of view little string theory on $R^{1,5}$ is dual to type IIA string theory on linear dilaton background [4]. Linear dilaton background is the one obtained by near horizon limit of NS5 brane. More precisely the supergravity solution for N NS5 branes is given by

$$\begin{aligned} ds^2 &= -dt^2 + d\vec{x}^2 + f(dr^2 + r^2 d\Omega_3^2), \\ e^{2\phi} &= {g_s}^2 f\,, \qquad dB = 2N\alpha'\epsilon_3\,, \qquad f = 1 + \frac{N\alpha'}{r^2}, \end{aligned} \tag{1.1}$$

where $d\vec{x}$ parameterizes a 5-dimensional flat space and ϵ_3 is the volume of $d\Omega^3$. In the decoupling limit where $g_s \to 0$ and l_s,$u = \frac{r}{g_s}$ are set to be finite, the N NS5 branes supergravity solution reads

$$\begin{aligned} ds^2 &= -dt^2 + d\vec{x}^2 + \frac{N\alpha'}{u^2}(du^2 + u^2 d\Omega_3^2), \\ e^{2\phi} &= \frac{N\alpha'}{u^2}\,, \qquad dB = 2N\alpha'\epsilon_3. \end{aligned} \tag{1.2}$$

This solution is called linear dilaton background which is conjectured to be dual to little string theory on $R^{1,5}$. Therefore by making use of AdS/CFT correspondence [5 − 7] (for a general review see [8]), one may use the gravity dual side to understand some features of little string theory side. Such studies have been done for the case where we have little string theory on $R^{1,5}$ [9 − 13]. The noncommutative deformation of this background and their corresponding Penrose limits have also been studied in [14 − 27].

Recently in the context of 1/2 BPS solutions [28] Lin and Maldacena have obtained the supergravity solution of type IIA N NS5 branes wrapped on S^5, that is [29]

$$
\begin{aligned}
ds^2_{10} &= N\alpha'\Big[-2r\sqrt{\frac{I_0}{I_2}}dt^2 + 2r\sqrt{\frac{I_2}{I_0}}d\Omega_5^2 + \sqrt{\frac{I_2}{I_0}}\frac{I_0}{I_1}(dr^2+d\theta^2) + \sqrt{\frac{I_2}{I_0}}\frac{I_0 I_1 s^2}{I_0 I_2 s^2 + I_1^2 c^2}d\Omega_2^2\Big]\\
B_2 &= N\alpha'\Big[\frac{-I_1^2 cs}{I_0 I_2 s^2 + I_1^2 c^2} + \theta\Big]d^2\Omega\\
e^{\Phi} &= g_0 N^{3/2} 2^{-1}\left(\frac{I_2}{I_0}\right)^{\frac{3}{4}}\left(\frac{I_0}{I_1}\right)^{\frac{1}{2}}\left(I_0 I_2 s^2 + I_1^2 c^2\right)^{-\frac{1}{2}}\\
C_1 &= -\alpha'^{1/2} g_0^{-1}\frac{1}{N}4r\frac{I_1}{I_2}(I_0^2 s^2 + I_1^2 c^2)dt\\
C_3 &= -\alpha'^{3/2} g_0^{-1}\frac{4I_0 I_1^2 s^3}{I_0 I_2 s^2 + I_1^2 c^2}dt\wedge d^2\Omega
\end{aligned}
\tag{1.3}
$$

where $I_n(r)$ are a series of modified Bessel functions of the first kind. Also s and c mean $\sin(\theta)$ and $\cos(\theta)$ respectively. This solution preserves 16 supercharges and has $R\times \mathrm{SO}(3)\times \mathrm{SO}(6)$ bosonic symmetry group. In our notation we have

$$
\begin{aligned}
d\Omega_5^2 &= d\theta_1^2 + \cos^2\theta_1 d\theta_2^2 + \sin^2\theta_1 d\Omega_3^2,\\
d\Omega_2^2 &= d\phi_1^2 + \cos^2\phi_1^2 d\phi_2^2.
\end{aligned}
\tag{1.4}
$$

In spirit of AdS/CFT one may suspect that type IIA string theory on this new background is dual to little string theory on $R^1\times S^5$. If this is the case one should first check whether there is a notion of decoupling limit. This can be done by making use of scattering of graviton from NS5 brane. Following [30] one can see that the scattering of transverse graviton will essentially lead to compute the scattering of a scalar field $\phi(r)$ the brane. This scalar field will satisfy the Laplace equation as follows

$$\partial_r(\sqrt{G}G^{rr}\partial_r\phi) + \partial_t(\sqrt{G}G^{tt}\partial_t\phi) = 0, \tag{1.5}$$

where G is the determinant of the metric. Setting $\phi = \alpha(r)\psi(r)e^{i\omega t}$ the above Laplace equation can be recast to the following Schrodinger like equation

$$\partial_r{}^2\psi - V(r) = 0, \tag{1.6}$$

where the potential is given by

$$V(r) = \frac{1}{2}\frac{A''}{A} - \frac{1}{4}\left(\frac{A'}{A}\right)^2 - \frac{I_2}{2rI_1}\omega^2, \tag{1.7}$$

with

$$A = r^3 I_1^2\,,\quad A' = \partial_r A\,,\quad A'' = \partial_r^2 A\,,\quad \frac{\alpha'(r)}{\alpha(r)} = \frac{-A'}{2A}\,. \tag{1.8}$$

The shape of the potential, having an infinite barrier, shows that the supergravity solution (1.3) is in the decoupling limit of NS5 branes, the limit for which the modes in the throat will decouple from the modes of the rest of the space. Therefore it is reasonable to say that there is a little string theory which lives on $R^1\times S^5$ that is dual to type IIA string theory on N NS5 branes wrapped on S^5 background, given by (1.3). So we can use the gravity side to obtain some information about little string theory side. For example one could study the Penrose limit of this solution and also following [31] we could consider semiclassical rotating and spinning closed string solutions on this background. This could give us an insight of what the little string theory on $R^1 \times S^5$ might be.

The paper is organized as follows. In section 2, following the study of localized rotating closed string in S^2, we will study the Penrose limit of this background and we will see that string theory on it, is exactly solvable. In section 3, we will study semiclassical closed strings rotating and spinning in subsequently S^2 and S^5. And the last section is devoted to conclusion.

2. Plane wave limit

As we said the background we are considering, (1.3), has $R \times \mathrm{SO}(3) \times \mathrm{SO}(6)$ bosonic symmetry. It means that the isometries of the metric are time, related to a conserved energy, three angle coordinates in S^5 which would lead to three spin conserved charges, and one angle coordinate in S^2 which would give a conserved angular momentum or R-symmetry charge. In the following two sections, according to these isometries, we will consider semiclassical closed strings that carry the corresponding conserved charges to probe this background and understand it better.

As the first case we will study a configuration in which the semiclassical closed strings are centered around the origin of S^5 sphere and stretched along radial coordinate, r. Also these folded closed strings rotate in one direction in S^2. In our notation the corresponding closed string configuration is given by

$$t = \kappa\tau\ , \quad r = r(\sigma)\ , \quad \varphi_2 = \nu\tau\ , \quad \theta = \frac{\pi}{2}\ , \tag{2.1}$$

and all the other coordinates are set to zero. Using this ansatz, the bosonic part of the superstring action is

$$S = \frac{-N}{4\pi}\int d\sigma d\tau\left(2r\sqrt{\frac{I_0}{I_2}}\kappa^2 + \sqrt{\frac{I_2}{I_0}}\frac{I_0}{I_1}r'^2 - \sqrt{\frac{I_2}{I_0}}\frac{I_1}{I_2}\nu^2\right), \tag{2.2}$$

where $r' = \partial_\sigma r$. The solution or ansatz considered here should also satisfy the Virasoro condition that is

$$r'^2 + \left(\frac{I_1^2}{I_0 I_2}\nu^2 - 2r\frac{I_1}{I_2}\kappa^2\right) = 0. \tag{2.3}$$

The corresponding conserved charges are

$$E = \frac{N\kappa}{\pi}\int d\sigma r\sqrt{\frac{I_0}{I_2}}\ , \qquad J = \frac{N\nu}{2\pi}\int d\sigma\sqrt{\frac{I_2}{I_0}\frac{I_1}{I_2}}. \tag{2.4}$$

Now the aim is to find the dependence of energy, E, on angular momentum, J. This can be done by making use of the Virasoro constraint. In general the Virasoro constraint can be thought of "zero energy" condition for a non-relativistic particle with a potential. We are looking for periodic solutions that satisfy this equation. But noting the fact that the first term, kinetic term, is positive definite, the potential, $V(r)$, should be negative or zero and simultaneously its derivative should be positive or zero.

In this ansatz the potential has no minimum or in other words the potential is always negative and has negative slope. Therefore we won't get any closed string solution except for $r = 0$ that is a zero size closed string solution. This will just happen when we have $\nu = 2\kappa$. Therefore in leading order, the relation between energy and angular momentum is given by

$$E = 2J. \tag{2.5}$$

What we have obtained are the states in which $E - 2J$ is finite. Now if we consider fluctuations around this classical solution, it would lead to the Penrose limit of the background. The idea is to consider the trajectory of a particle that is moving very fast along one direction in S^2 and to focus on the geometry that this particle sees. These fluctuations are

$$\begin{aligned} t &= \kappa\tau + \frac{\tilde{t}}{2^{1/4}\sqrt{N}} , \quad & \varphi_2 &= 2\kappa\tau + \frac{\tilde{\varphi}_2}{2^{1/4}\sqrt{N}} , \quad & r &= \frac{2^{1/4}\tilde{r}}{\sqrt{N}} , \\ \theta &= \frac{\pi}{2} + \frac{2^{1/4}y}{\sqrt{N}} , & \varphi_1 &= \frac{z}{2^{1/4}\sqrt{N}} , & \Omega &= \tilde{\Omega} . \end{aligned} \tag{2.6}$$

We have also imposed the rescaling of coordinates in these relations to get the leading finite terms in the sigma model action when N is large.

The bosonic part of the string sigma model action, in general background, is

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} \left[\left(g^{ab} G_{\mu\nu}(X) + \epsilon^{ab} B_{\mu\nu}(X) \right) \partial_a X^\mu \partial_b X^\nu + \frac{1}{2}\alpha' \Phi R^{(2)} \right], \tag{2.7}$$

where $R^{(2)}$ is worldsheet curvature and since in our case the metric is η^{ab}, $R^{(2)}$ is zero. Using the fluctuations (2.6) the first term of the string action for the metric (1.3) reads

$$\begin{aligned} S_1 = \frac{-1}{4\pi} \int d\sigma d\tau \Big[& \partial_a y \partial^a y + \partial_a z \partial^a z + \partial_a X_i \partial^a X^i + \kappa^2 (X_i X^i + 16 y^2 + 4 z^2) \\ & - 4 \partial_a \tilde{t} \partial^a \tilde{t} + \partial_a \tilde{\varphi}_2 \partial^a \tilde{\varphi}_2 \Big], \end{aligned} \tag{2.8}$$

where the angular coordinates in S^5 and radial coordinate, r, are shown by X_i where $i = 1, \ldots, 6$ (6-dimensional flat space). On the other hand using the definition of B field in (1.3) the second term of the action would be

$$S_2 = \frac{-1}{2\pi} \int d\sigma d\tau \; 3\kappa y \partial_\sigma z. \tag{2.9}$$

Let us define the lightcone coordinates as

$$x^+ = t + \frac{\varphi_2}{2} , \qquad x^- = t - \frac{\varphi_2}{2}, \tag{2.10}$$

therefore

$$x^+ = 2\kappa\tau + \frac{1}{2^{1/4}\sqrt{N}} \left(\tilde{t} + \frac{1}{2}\tilde{\varphi}_2 \right) , \qquad x^- = \frac{1}{2^{1/4}\sqrt{N}} \left(\tilde{t} - \frac{1}{2}\tilde{\varphi}_2 \right) . \tag{2.11}$$

If we perform the following rescaling for x^-

$$x^- = \frac{\tilde{x}^-}{\sqrt{2}N}. \tag{2.12}$$

and use the fact that $\kappa = \frac{1}{2}\partial_\tau x^+$, lightcone gauge, the sigma model action is written as

$$S = \frac{-1}{4\pi}\int d\sigma d\tau\Big[\partial_a X_i \partial^a X^i + \frac{1}{4}\partial_\tau x^+ \partial_\tau x^+ (X_i X^i + 16y^2 + 4z^2)$$
$$+\partial_a y \partial^a y + \partial_a z \partial^a z - 4\partial_a x^+ \partial^a \tilde{x}^- + 3y\partial_\tau x^+ \partial_\sigma z\Big]. \tag{2.13}$$

This action is the one for eight massive bosons, related to eight transverse coordinates. Therefore by using it we can see that the fluctuations around closed zero size strings will see a plane wave background given by the following metric

$$ds^2 = \alpha'\Big[dy^2 + dz^2 + dX_6^2 - \frac{1}{4}(X_6^2 + 16y^2 + 4z^2)dx^{+2} - 4dx^+ d\tilde{x}^-\Big], \tag{2.14}$$

and B field

$$B_{+z} = 3\alpha' y. \tag{2.15}$$

Under this change of coordinates dilaton field will remain finite if we rescale g_0 by $N^{-\frac{3}{2}}$. Also the RR field strengths corresponding to C_1 and C_3 vanish when $N \to \infty$.

By making use of the action we can write the equations of motion for the strings moving on this plane wave background as

$$\partial_a\partial^a y - 3\kappa\partial_\sigma z - 16\kappa^2 y = 0\,, \quad \partial_a\partial^a z + 3\kappa\partial_\sigma y - 4\kappa^2 z = 0$$
$$\partial_a\partial^a X_i - \kappa^2 X_i = 0\,, \quad i = 1,\dots,6. \tag{2.16}$$

These equations of motion are similar to what we have for harmonic oscillator, six independent and two coupled ones. Therefore string theory is exactly solvable on the plane wave background (2.14). The results for normal oscillating modes are

$$\omega_n = \kappa\sqrt{1 + \frac{n^2}{\kappa^2}}\,, \qquad \omega^\pm = \kappa\sqrt{10 + \frac{n^2}{\kappa^2} \pm 3\sqrt{4 + \frac{n^2}{\kappa^2}}}. \tag{2.17}$$

Now we are able to obtain energy, E, and angular momentum, J, relations for the strings moving in this plane wave background. For the classical closed string case we had $E = 2J$. But when we consider quantum fluctuations it would be

$$E - 2J = \frac{1}{2\pi}\int d\sigma\Big(\kappa(X_i X^i + 16y^2 + 4z^2) + 4\sqrt{N}2^{\frac{1}{4}}\partial_\tau\tilde{x}^- + 3y\partial_\sigma z\Big). \tag{2.18}$$

On the other hand we can write the Virasoro condition when we consider these fluctuations, that is

$$(\partial_\tau X_i \partial_\tau X^i + \partial_\sigma X_i \partial_\sigma X^i) + (\partial_\tau y \partial_\tau y + \partial_\sigma y \partial_\sigma y) + (\partial_\tau z \partial_\tau z + \partial_\sigma z \partial_\sigma z)$$
$$-\kappa^2(X_i X^i + 16y^2 + 4z^2) - 4(\partial_\tau\tilde{t}\partial_\tau\tilde{t} + \partial_\sigma\tilde{t}\partial_\sigma\tilde{t}) + (\partial_\tau\tilde{\varphi}_2\partial_\tau\tilde{\varphi}_2 + \partial_\sigma\tilde{\varphi}_2\partial_\sigma\tilde{\varphi}_2)$$

$$-8\kappa 2^{\frac{1}{4}}\sqrt{N}\partial_\tau \tilde{x}^- = 0 \tag{2.19}$$

Using this Virasoro condition we can replace the term, $8\kappa 2^{\frac{1}{4}}\sqrt{N}\partial_\tau \tilde{x}^-$, in (2.18). The result is

$$E - 2J = H_{LightCone}, \tag{2.20}$$

where $H_{L.C.}$ is the hamiltonian of the system in light cone gauge and is given by

$$\begin{aligned} E - 2J = & \frac{1}{4\pi\kappa}\int d\sigma \Big((\partial_\tau X_i \partial_\tau X^i + \partial_\sigma X_i \partial_\sigma X^i) + (\partial_\tau y \partial_\tau y + \partial_\sigma y \partial_\sigma y) \\ & + (\partial_\tau z \partial_\tau z + \partial_\sigma z \partial_\sigma z) + \kappa^2 (X_i X^i + 16 y^2 + 4z^2) - 4(\partial_\tau \tilde{t}\partial_\tau \tilde{t} + \partial_\sigma \tilde{t}\partial_\sigma \tilde{t}) \\ & + (\partial_\tau \tilde{\varphi}_2 \partial_\tau \tilde{\varphi}_2 + \partial_\sigma \tilde{\varphi}_2 \partial_\sigma \tilde{\varphi}_2) + 6\kappa y \partial_\sigma z \Big). \end{aligned} \tag{2.21}$$

Therefore by making use of (2.17) we have

$$\begin{aligned} E - 2J = \sum_n & N_n^{(6)}\sqrt{1 + \frac{8N^2 n^2}{J^2}} + N_n^+ \sqrt{10 + \frac{8N^2n^2}{J^2} + 3\sqrt{4 + \frac{8N^2n^2}{J^2}}} \\ & + N_n^- \sqrt{10 + \frac{8N^2n^2}{J^2} - 3\sqrt{4 + \frac{8N^2n^2}{J^2}}}, \end{aligned} \tag{2.22}$$

where $N_n^\pm$ is the occupation number along z and y and also N_n^i are occupation numbers along six dimensional flat space, characterized by X_i. In the classical case we saw that the closed string satisfied the relation, $E - 2J = 0$. But now we can see this relation will get some corrections which is controlled by $\frac{N^2}{J^2}$.

Using AdS/CFT correspondence, there should be some operators in little string theory on $R^1 \times S^5$, for which both E and J are large but $E - 2J$ is finite. These states are not BPS, because $E - 2J$ gets quantum corrections, though these corrections are under control and in fact the expansion parameter is given by $\frac{N^2}{J^2}$. Note also that $\frac{N^2}{J^2}$ remains finite in the $N \to \infty$ limit. To summarize one may conjecture that type IIA string theory on plane wave background (2.14), is dual to operators in little string theory on $R^1 \times S^5$ whose E and J are large but $E - 2J$ is finite and is given by (2.22).

3. Rotating and spinning closed strings

In this section we will consider semiclassical closed strings stretched along radius and rotate in S^5 and S^2, each in one direction. The ansatz that describes this would be

$$t = \kappa\tau\ , \quad r = r(\sigma)\ , \quad \varphi_2 = \nu\tau\ , \quad \theta_2 = \omega\tau\ , \quad \theta = \frac{\pi}{2}\ , \tag{3.1}$$

and other coordinates are set to zero. For this case the action is written as

$$S = \frac{-N}{4\pi}\int d\sigma\, d\tau \left(2r\sqrt{\frac{I_0}{I_2}}\kappa^2 + \sqrt{\frac{I_2}{I_0}}\frac{I_0}{I_1}r'^2 - 2r\sqrt{\frac{I_2}{I_0}}\omega^2 - \sqrt{\frac{I_2}{I_0}}\frac{I_1}{I_2}\nu^2 \right). \tag{3.2}$$

This ansatz will also satisfy the Virasoro condition that is

$$r'^2 - \frac{r^2}{4}\frac{I_0 - I_2}{I_0 I_2}(4\kappa^2 - \nu^2)\left[I_0 - I_2 \frac{4\omega^2 - \nu^2}{4\kappa^2 - \nu^2}\right] = 0. \tag{3.3}$$

The isometries of the action will result in the following conserved charges

$$E = \frac{N\kappa}{\pi}\int d\sigma r\sqrt{\frac{I_0}{I_2}}\,,\qquad S = \frac{N\omega}{\pi}\int d\sigma r\sqrt{\frac{I_2}{I_0}}\,,\qquad J = \frac{N\nu}{2\pi}\int d\sigma\sqrt{\frac{I_2}{I_0}}\frac{I_1}{I_2}. \tag{3.4}$$

The aim of probing the background using strings is to obtain the dependence of E on S and J, for generic κ, ω and ν. In the first step, using the relations written in above equation we will have

$$E = \frac{4\kappa}{\nu}J + \frac{\kappa}{\omega}S. \tag{3.5}$$

To see whether this choice has a solution or not, we can use Virasoro constraint. As what was done before, by looking at the potential we see that the periodicity condition will be satisfied if we consider $2\kappa > \nu$ and $2\omega > \nu$. The turning point is r_0 and is given by

$$\frac{I_0(r_0)}{I_2(r_0)} = \frac{4\omega^2 - \nu^2}{4\kappa^2 - \nu^2}. \tag{3.6}$$

It is very difficult to obtain solutions precisely. So we will study its long and short string limits. If we define

$$1 + \eta = \frac{4\omega^2 - \nu^2}{4\kappa^2 - \nu^2}, \tag{3.7}$$

therefore $\eta \to 0$ and $\eta \to \infty$ correspond to long, $r \to \infty$, and short, $r \to 0$, limits, respectively. Now we are ready to study short and long closed strings rotating in this supergravity background.

3.1 Short strings

In the short string limit, $r_0 \to 0$, we can check that whether there is a periodic solution. To see this we expand (3.3) for small $r \to 0$ we get

$$r'^2 \approx 2(4\kappa^2 - \nu^2) - \frac{1}{4}(4\kappa^2 - \nu^2)\left(\frac{2}{3} + \eta\right) r^2, \tag{3.8}$$

and this equation will be satisfied by a periodic solution for r that is $r = r_0 \sin\sigma$, if we have

$$r_0 = \sqrt{2(4\kappa^2 - \nu^2)}\,,\quad \frac{1}{4}(4\kappa^2 - \nu^2)\left(\frac{2}{3} + \eta\right) = 1. \tag{3.9}$$

Therefore using the definition of η and the fact that in short string case $\eta \to \infty$ we obtain

$$4\kappa^2 - \nu^2 \sim \frac{4}{\eta}\,,\qquad 4\omega^2 - \nu^2 \sim \frac{4}{\eta} + \eta. \tag{3.10}$$

Now using the relation obtained for angular momentum J and expanding it for $r \to 0$, the dependence of J on ν will be

$$J \sim \sqrt{2}N\nu. \tag{3.11}$$

Also if we do the same for S we get

$$S \sim \frac{2\sqrt{2}N\omega}{\eta}. \tag{3.12}$$

Plugging this equation into (3.10) we get an expression for ω at leading order, which can be used to obtain $\frac{1}{\eta}$ as follows

$$\frac{1}{\eta} \sim \frac{S}{\sqrt{2}N\sqrt{4+\frac{J^2}{2N^2}}}. \tag{3.13}$$

We can substitute the above equation in (3.10) and find the final results for κ and ω as

$$4\kappa^2 \sim \frac{J^2}{2N^2} + \frac{4S}{\sqrt{2}N}\frac{1}{\sqrt{4+\frac{J^2}{2N^2}}}, \tag{3.14}$$

$$4\omega^2 \sim 4 + \frac{J^2}{2N^2} + \frac{2\sqrt{2}S}{N}\frac{1}{\sqrt{4+\frac{J^2}{2N^2}}}. \tag{3.15}$$

We need to find the dependence of E on S and J. Using (3.5) and the relations for ω, ν and κ the final result will be

$$E \approx \sqrt{\frac{2J^2}{N^2} + \frac{16\sqrt{2}S}{N\sqrt{16+\frac{2J^2}{N^2}}}}\left(\sqrt{2}N + \frac{S}{\sqrt{16+\frac{2J^2}{N^2}+\frac{16\sqrt{2}S}{N\sqrt{16+\frac{2J^2}{N^2}}}}}\right). \tag{3.16}$$

For the case where both S and J are small we get

$$E^2 \approx 8\sqrt{2}NS + 4(J^2+S^2), \tag{3.17}$$

which actually represent Regge trajectories in the flat space. The other limit is $S \gg J$ for which energy is given by

$$E \approx S - \frac{\sqrt{2}}{16N}J^2 - \sqrt{2}N + \sqrt{S}\left(2^{\frac{7}{4}}\sqrt{N} - \frac{1}{16}\left(\frac{\sqrt{2}}{N}\right)^{\frac{3}{2}}J^2\right). \tag{3.18}$$

Also when $S \ll J$ we can find energy as

$$E \approx 2J + S + \frac{4N^2S}{J^2} - \frac{8N^4S}{J^4}. \tag{3.19}$$

It is easily seen that this expression for energy is an expansion in terms of $\frac{N^2}{J^2}$ and is related to the leading quantum term in the spectrum of string on plane wave background, (2.22).

3.2 Long strings

As we saw the long string limit corresponds to $r_0 \to \infty$ and $\eta \to 0$. In (3.3) we expand r'^2 for $r \to \infty$, therefore we find

$$r'^2 \approx (3\kappa^2 - \nu^2 + \omega^2) + 2r(\kappa^2 - \omega^2). \tag{3.20}$$

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Considering the fact that r_0 is the turning point ($r' = 0$), we obtain

$$r_0 = \frac{2}{\eta} + \frac{1}{2}. \tag{3.21}$$

On the other hand we note that $r' = \frac{dr}{d\sigma}$ and also for $0 < \sigma < \frac{\pi}{4}$ the function $r(\sigma)$ increases from zero to a maximal value r_0, so

$$2\pi = \int_0^{2\pi} d\sigma \approx \frac{4}{\sqrt{2(\omega^2 - \kappa^2)}} \int_0^{r_0} \frac{dr}{\sqrt{\frac{2}{\eta} + \frac{1}{2} - r}}. \tag{3.22}$$

Therefore we get

$$\omega^2 - \kappa^2 \sim \frac{4}{\pi^2}\left(\frac{4}{\eta} + 1\right), \tag{3.23}$$

and using the definition of η we find

$$4\kappa^2 - \nu^2 \sim \frac{16}{\pi^2 \eta}\left(\frac{4}{\eta} + 1\right). \tag{3.24}$$

Now if we expand the definition of J and S for large r we get that

$$J \sim N\nu\,, \qquad S \sim \frac{2N\omega}{3}\left(\frac{4}{\eta} - 2\right), \tag{3.25}$$

where we have used (3.4). In this step we should try to obtain a relation for η dependence on S and J. To do this we use (3.23) and (3.24) to find a relation for ω in terms of η and ν. Now if we substitute the result in the equation for S (3.25), we get a second order equation for η that is

$$S + \frac{49N}{12\pi} - \frac{\pi J^2}{12N} - \frac{4N}{3\pi\eta}\left(\frac{8}{\eta} + 1\right) \approx 0, \tag{3.26}$$

which can be solved to find

$$\frac{1}{\eta} \approx \frac{1}{16}\left(-1 + \sqrt{99 + \frac{24\pi S}{N} - \frac{2\pi^2 J^2}{N^2}}\right). \tag{3.27}$$

We substitute it in equations (3.23) and (3.24) and get the final results for κ and ω that are

$$4\omega^2 \sim \frac{J^2}{2N^2} + \frac{6S}{\pi N} + \frac{36}{\pi^2} + \frac{9}{2\pi^2}\left(\sqrt{99 + \frac{24\pi S}{N} - \frac{2\pi^2 J^2}{N^2}}\right), \tag{3.28}$$

$$4\kappa^2 \sim \frac{J^2}{2N^2} + \frac{6S}{\pi N} + \frac{24}{\pi^2} + \frac{1}{2\pi^2}\left(\sqrt{99 + \frac{24\pi S}{N} - \frac{2\pi^2 J^2}{N^2}}\right). \tag{3.29}$$

Now we can use the equation obtained for energy, (3.5), and obtain energy in terms of angular momentum and spin. Energy in the limit where S is large and J is small is given by

$$E \approx \frac{14N}{3\pi} + S + \frac{4}{3}\sqrt{\frac{6NS}{\pi}} + \frac{55\sqrt{6}}{36}\left(\frac{N}{\pi}\right)^{\frac{3}{2}} \frac{1}{\sqrt{S}} + \sqrt{\frac{\pi}{6N}}\frac{J^2}{\sqrt{S}}. \tag{3.30}$$

We can see that in contrast to similar cases in $Ads_5 \times S^5$ [31, 32], the corrections to energy are not in terms of $\ln S$, but in terms of $\sqrt{S}$. Of course this might be understood from the fact that in the $AdS_5 \times S^5$ case we are dealing with a gauge theory. It should also be mentioned that limits in which J is large or J is greater than S in this long string limit are not reasonable.

4. Conclusion

In this paper we have studied the 1/2 BPS geometry of N NS5 branes wrapped on S^5 using folded closed string probes. We have first considered a point like closed string configuration which rotates in S^2. The string sigma model expansion around this classical solution resulted in pp wave limit of this background. Such a result should be compared to pp wave limit of NS5 branes in $R^{1,5}$ case. The pp wave metric we have obtained for N NS5 branes wrapped on S^5, is 10-dimensional but for the case in which NS5 brane is defined on $R^{1,5}$ the result is 4-dimensional plane wave times 6-dimensional flat space [24 − 27].

In the case we have considered, little string theory on $R \times S^5$ is dual to type IIA string theory on N NS5 branes wrapped on S^5 background. Actually one may interpret the strings in little string theory as D2 branes stretched between two NS5 branes in type IIA string theory. In the limit where NS5 branes approach each other we will have to deal with tensionless strings in little string theory side. Now using the duality one may deduce that little string theory on $R \times S^5$ is composed of closed strings. Because on S^5 all dimensions are compact and therefore periodic. This is in contrast with having just open strings in little string theory on $R^{1,5}$.

We also considered short and long string limits of folded closed strings, centered around the origin and stretched along the radial coordinate. Such a configuration rotates along one direction in S^2 and also spins along one direction in S^5. Such a configuration would result in a state in little string theory that has spin. It should be mentioned that such states do not exist in little string theory on $R^{1,5}$. Because there are not such classical closed string configurations in its supergravity dual. In the short string case as expected we obtained Regge trajectory as in the flat space when S and J are small. In long string limit we have also seen that corrections to energy are in terms of $\sqrt{S}$.

One could also consider the most general case in which the folded closed string rotates in one direction in S^2 and spins in three directions in S^5. Such a configuration that is called multi-spin string solution, carries all the conserved charges related to the isometries of the metric [33].

One may study the background by probing it with NS5 brane wrapped on S^5 or D2 brane wrapped on S^2. Such cases could be compared to giant gravitons in $AdS_5 \times S^5$, where two different D3 branes could wrap over two 3-spheres of $AdS_5 \times S^5$ in global coordinates. It would also be interesting to study the genuine dynamics of stings such as splitting in this background. Such analysis has been done for $AdS_5 \times S^5$ background, for example, in [34, 35].

Finally we note that since the string theory on plane wave background (2.14) is exactly solvable, one could also study open string which would result in different D-brane configurations that might exist in this background. In this paper we have only studied the bosonic part of the string sigma model action. One may also consider the fermionic part. Such study has been done in the case of little string theory living on $R^{1,5}$ [27].

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